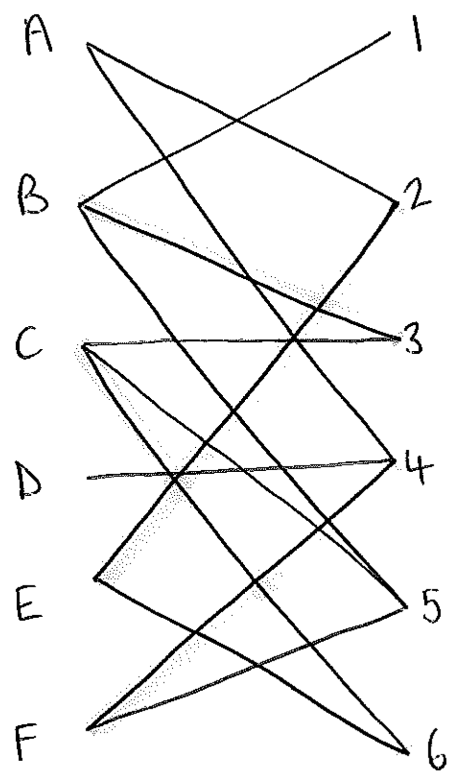
Jun '07

1a)



b) because D can only be matched to 4 so if F gets matched to 4 a complete matching is impossible

c) A − 2 + E − 6 + C − 3 + B − 1

D − 4 + F − 5

A2
B1
C3
D4
E6
F5

2a) 28 22 20 17 14 11 6 5

28 22 20 17 | 14 11 6 5

28 22 | 20 17 | 14 11 | 6 5

28 | 22 | 20 | 17 | 14 | 11 | 6 | 5

14 11 6 5 28 22 20 17

14 6 28 20

11 5 22 17

6 5 14 11 20 17 28 22

5 6 11 14 17 20 22 28

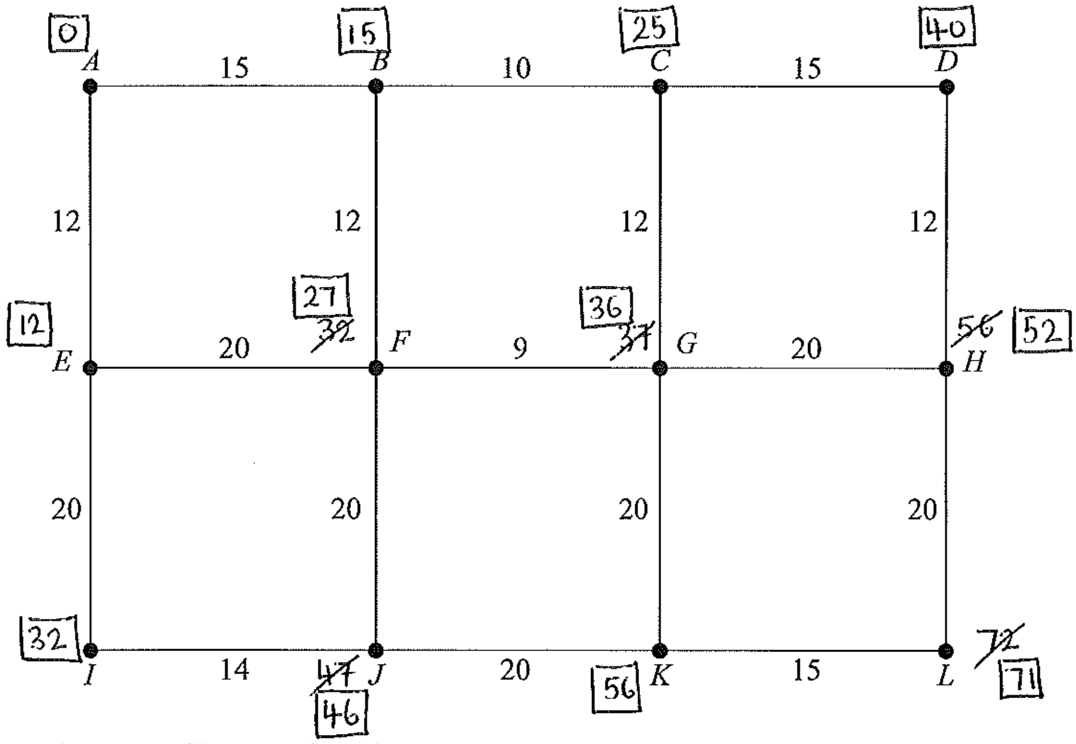
bi) Comparisons = 4

ii) swaps = 4

c) 7 + 6 + 5 + 4 + 3 + 2 + 1 = 28

3 [**Figure 1**, printed on the insert, is provided for use in this question.]

The following network represents the footpaths connecting 12 buildings on a university campus. The number on each edge represents the time taken, in minutes, to walk along a footpath.



(a) (i) Use Dijkstra's algorithm on **Figure 1** to find the minimum time to walk from A to L. *(7 marks)*

(ii) State the corresponding route. *(1 mark)*

(b) A new footpath is to be constructed. There are two possibilities:

from A to D, with a walking time of 30 minutes; or

from A to I, with a walking time of 20 minutes.

Determine which of the two alternative new footpaths would reduce the walking time from A to L by the greater amount. *(3 marks)*

Turn over ▶

3 ai) see sheet 71 mins

ii) A B F G K L

b) AD = 30 $\xrightarrow[12]{}$ H $\xrightarrow[20]{}$ L = 62

AI = 20 $\xrightarrow[14]{}$ J $\xrightarrow[20]{}$ K $\xrightarrow[15]{}$ L = 69

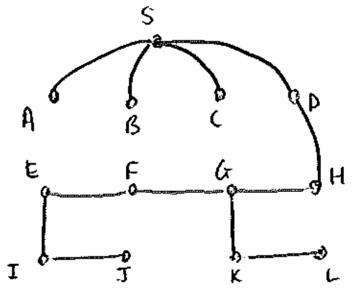
New path AD by 9 mins

4ai)

SD	12
SC	13
SA	14
SB	16
DH	75
HG	23
GF	22
FE	24
EI	81
IJ	12
GK	83
KL	16
	391

ii) 391

iii)



iv) FG 22

GH 23

b) odd vertices E, H, J, K

EH = 69 (EFGH)

EJ = 93 (EIJ)

EK = 129 (EFGK)

JK = 131

HK = 106 (HGK)

HJ = 142 (HGFJ)

EH + JK = 69 + 131 = 200

EJ + HK = 93 + 106 = 199

EK + HJ = 129 + 142 = 271

1135 + 199 = 1334

5a) $5x + 10y \leq 1500 \Rightarrow x + 2y \leq 300$

$32x + 8y \leq 4000 \Rightarrow 4x + y \leq 500$

$\left.\begin{array}{l} x \geq 50 \\ y \geq 50 \end{array}\right\}$ at least 50 of each

$x + y \geq 140$ } at least 140 in total

bi) see next sheet

ii) $P = 0.8x + 1.2y$

max at (100, 100)

$P = 0.8(100) + 1.2(100)$

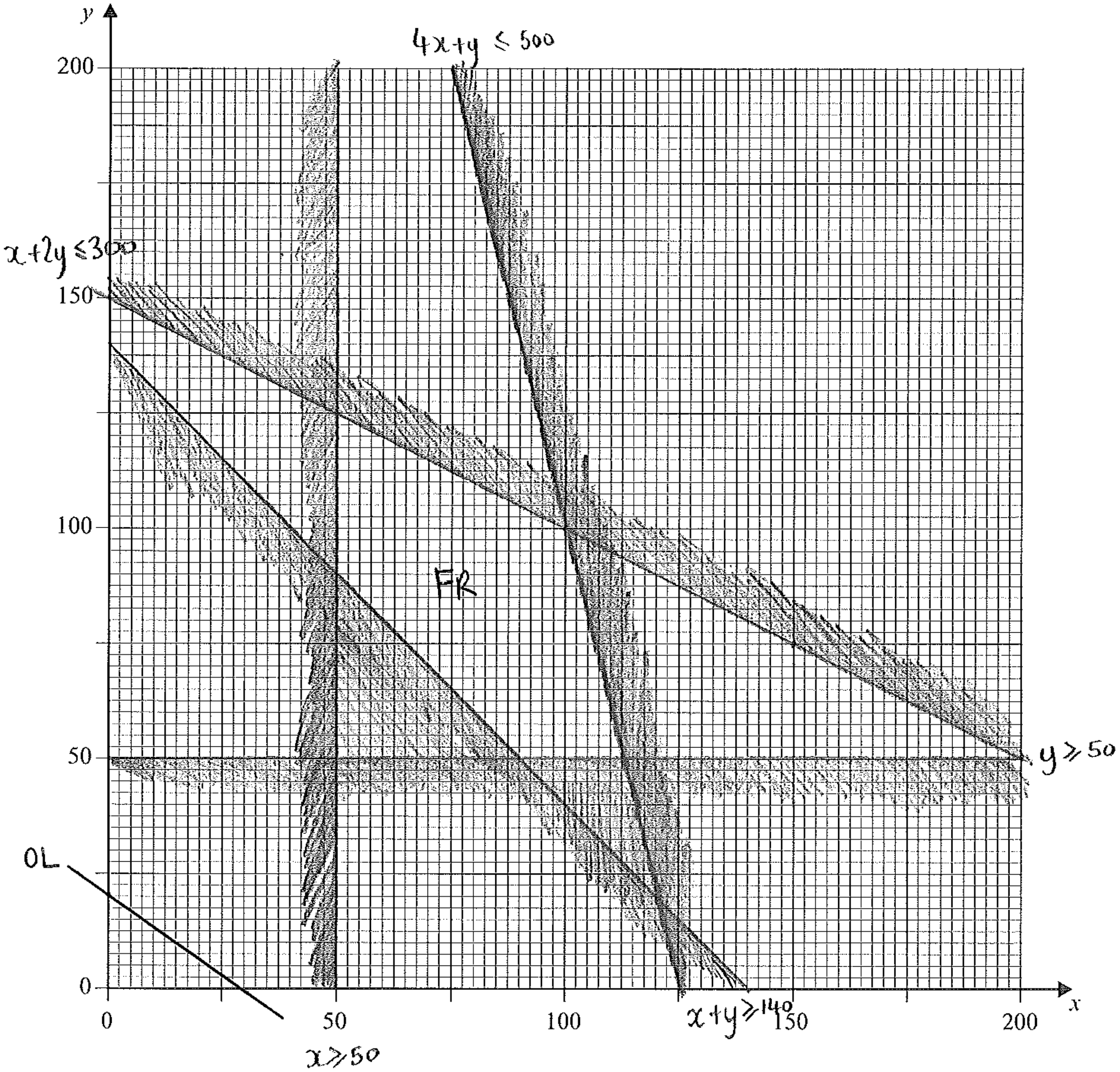
$= 80 + 120 = £200$

iii) min at (90, 50)

$P = 0.8(90) + 1.2(50)$

$= 72 + 60 = £132$

Figure 2 (for use in Question 5)



$P = 0.8x + 1.2y$

$240 = 8x + 12y$

$(0,20)$ $(30,0)$

6ai) $G \to P \to A \to N \to R \to G$

65 115 155 125 160 = 620

ii)

PA	115
RN	125
NA	155
	395

GP	65
GR	160
	225

$395 + 225 = 620$

iii) As the upper and lower bound are the same, 620 must be the optimal solution.

bi) $G \to I \to M \to V \to G$

20 32 21 19 = 92

ii) $G \to V \to M \to I \to G$

30 24 18 15 = 87

iii) $3 \times 2 \times 1 = 6$

iv) $n!$